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Model Dependence of the Matrix Elements of the $\Delta B=2$ Six-Quark Operators in Neutron-Antineutron Transitions

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ABSTRACT

To investigate their model dependence, the matrix elements of the six independent $\Delta B=2$ six-quark operators, which enter in neutron-antineutron transitions, were evaluated using the neutron wave functions in the Relativistic Potential Model and in the Harmonic Oscillator Shell Model. The results were compared with the corresponding values previously calculated in the MIT Bag Model and the matrix elements were found to be very sensitive to the model used for the neutron wave function, some differing not only by more than an order of magnitude but also in sign.

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Baryon number nonconservation is one of the most remarkable consequences of grand unified theories of the strong, electromagnetic and weak interactions. In a class of grand unified theories, neutron-antineutron transitions[1,2,3,4] are predicted in addition to the processes which conserve B-L, such as nucleon decay and hydrogen-antihydrogen transitions. The analysis of free $n \leftrightarrow \bar{n}$ transitions in terms of the (lowest) dimension 9 six-quark operators, has been given by Kuo and Love[4], Rao and Shrock[5] and Caswell, Milutinovic and Senjanovic[6]. In addition, Kuo and Love[4] have pointed out the importance of $SU(2)_L \times U(1)$ symmetry breaking at an intermediate mass scale of 10^2-10^5 GeV which enhances free $n \leftrightarrow \bar{n}$ transitions to an observable level.

More recently, Rao and Shrock[5] have evaluated the matrix elements of the general set of dimension 9 six-quark operators that can contribute to $n \leftrightarrow \bar{n}$ transitions using the neutron wave function in the MIT Bag Model[7]. To investigate the sensitivity of the values of these $\Delta B=2$, six-quark operators, we have calculated the same matrix elements using two phenomenoligically plausible alternatives to the MIT Bag Model: The Relativistic Potential Model[8] and the Harmonic Oscillator Shell Model[9]. Our results, presented below, when compared with the previous calculations using the MIT Bag Model, demonstrate very clearly that the nucleon-antinucleon matrix elements of these $\Delta B=2$, six-quark operators are quite model dependent. While it is obvious that all such hadron matrix elements of quark operators are model dependent, the quantitative comparison of the results obtained in the various models for these $\Delta B=2$, six-quark operators is rather striking. The magnitudes of the matrix elements vary by more than an order of magnitude among the models and even differences in sign occur.

The characteristic $n \leftrightarrow \bar{n}$ transition time for free neutrons is given by

$$\tau^{-1}_{n\bar{n}} = \langle \bar{n} \,|-\int d^3x \, L_{eff}\,| n\rangle \qquad (1)$$

where L_{eff} is a linear combination of the u and d six-quark operators $(0m)_\chi$ which have dimension 9:

$$L_{eff} = \sum_{m,\chi} C_{m,\chi} \, (0_m)_\chi + \text{h.c.} \qquad (2)$$

The compound helicity of the six-quark operator $(0_m)_\chi$ is denoted by χ, m=1,2,3 is an integer label and $C_{m,\chi}$ are c-number coefficients. Following the notation of Rao and Shrock[5] the operators $(0_m)_{\chi_1\chi_2\chi_3}$ can conveniently be expressed in terms of u and d quark operators as follows:

$$(0_1)_{\chi_1\chi_2\chi_3} = \{(u^{T\alpha}_{\chi_1} \, C \, u^{\beta}_{\chi_1})(d^{T\gamma}_{\chi_2} \, C \, d^{\delta}_{\chi_2})(d^{T\rho}_{\chi_3} \, C \, d^{\sigma}_{\chi_3})\}(T_S)_{\alpha\beta\gamma\delta\rho\sigma} \qquad (3)$$

$$(0_2)_{\chi_1\chi_2\chi_3} = \{(u^{T\alpha}_{\chi_1} \, C \, d^{\beta}_{\chi_1})(u^{T\gamma}_{\chi_2} \, C \, d^{\delta}_{\chi_2})(d^{T\rho}_{\chi_3} \, C \, d^{\sigma}_{\chi_3})\}(T_S)_{\alpha\beta\gamma\delta\rho\sigma} \qquad (4)$$

$$(0_3)_{\chi_1\chi_2\chi_3} = \{(u^{T\alpha}_{\chi_1} \, C \, d^{\beta}_{\chi_1})(u^{T\gamma}_{\chi_2} \, C \, d^{\delta}_{\chi_2})(d^{T\rho}_{\chi_3} \, C \, d^{\sigma}_{\chi_3})\}(T_A)_{\alpha\beta\gamma\delta} \qquad (5)$$

Here χ_i indicates left(L) or right(R) helicity, C is the charge conjugation matrix, u and d denote quark fields and α, β, etc., are color indices. The dependence on color is given by the tensors

$$(T_S)_{\alpha\beta\gamma\delta\rho\sigma} = \varepsilon_{\rho\alpha\gamma}\,\varepsilon_{\sigma\beta\delta} + \varepsilon_{\sigma\alpha\gamma}\,\varepsilon_{\rho\beta\delta} + \varepsilon_{\rho\beta\gamma}\,\varepsilon_{\sigma\alpha\delta} + \varepsilon_{\sigma\beta\gamma}\,\varepsilon_{\rho\alpha\delta} \qquad (6)$$

and

$$(T_A)_{\alpha\beta\gamma\delta\rho\sigma} = \varepsilon_{\rho\alpha\beta}\,\varepsilon_{\sigma\gamma\delta} + \varepsilon_{\sigma\alpha\beta}\,\varepsilon_{\rho\gamma\delta} \qquad (7)$$

The operators $(0_m)_\chi$ are not all linearly independent due to the symmetry relations:[5,6]

$$(0_1)_{\chi_1 LR} = (0_1)_{\chi_1 RL} \qquad (8)$$

$$(0_m)_{LR\chi_3} = (0_m)_{RL\chi_3} \qquad (9)$$

and

$$(O_2)_{\chi_1\chi_1\chi_3} - (O_1)_{\chi_1\chi_1\chi_3} = 3(O_3)_{\chi_1\chi_2\chi_3} \tag{10}$$

The four-component quark wave function in the ground state of the quark confinement models of interest is of the usual form

$$q(r) = \begin{bmatrix} f(r) \\ \vec{\sigma}\cdot\hat{r}\ g(r) \end{bmatrix} \chi \tag{11}$$

where f(r) and g(r) are determined by the particular model. Specifically we shall consider the Relativistic Potential Model[8] and the Harmonic Oscillator Shell Model[9] for comparison with the MIT Bag Model[7]. The n-$\bar{n}$ matrix elements $\langle O_m\rangle_\chi = \langle n|(O_m)_\chi|\bar{n}\rangle$, which Rao and Shrock[5] calculated for the MIT Bag Model, can readily be expressed in terms of only three independent integrals $I_{a,b,c}$ in all these models:

$$\begin{aligned}
\langle O_1\rangle_{RRR} &= 24\ (I_a - I_b) \\
\langle O_1\rangle_{LLR} &= 8\ (3I_a - I_b) \\
\langle O_1\rangle_{RLL} &= 8\ (3I_a + 5I_b) \\
\langle O_2\rangle_{RRR} &= -6(I_a - I_b) \\
\langle O_2\rangle_{LLR} &= 2\ (-3I_a + 7I_b) \\
\langle O_2\rangle_{RLL} &= -2\ (3I_a + 5I_b) \\
\langle O_3\rangle_{RRR} &= -10(I_a - I_b) \\
\langle O_3\rangle_{LRR} &= \frac{2}{3}\ (3I_a + 5I_b) + 12I_c \\
\langle O_3\rangle_{LRR} &= \frac{2}{3}\ (3I_a - 7I_b) - 12I_c
\end{aligned} \tag{12}$$

In terms of the upper and lower components $f(r)$ and $g(r)$ of the wave function $q(r)$, the integrals $I_{a,b,c}$ are

$$I_a = \int d^3r[g^2(r)-f^2(r)]^3 \tag{13}$$

$$I_b = 4\int d^3r\ f(r)^2\ g(r)^2\ [f(r)^2 - g^2(r)] \tag{14}$$

and

$$I_c = \int d^3r\ [f^2(r) + g^2(r)]^2\ [g^2(r) - f^2(r)] \tag{15}$$

Using the neutron wave functions in the Relativistic Potential Model and in the Harmonic Oscillator Shell Model the numerical values of the integrals $I_{a,b,c}$ were computed. The results are presented in Table I, together with the values for the MIT Bag Model[5], for comparison. And Table II gives the corresponding numerical results for the neutron-antineutron matrix elements of the operators $(O_m)_\chi$ in the same models: the Relativistic Potential Model, the Harmonic Oscillator Shell Model and, for comparison, the MIT Bag Model values previously calculated[5].

It is quite apparent from the Tables that these n-$\bar{n}$ matrix elements of the six $\Delta B=2$ six-quark operators $(O_m)_\chi$ are <u>very</u> model dependent. The magnitude drastically changes from model to model and even the signs of some of the matrix elements are different in the different models.

It is interesting to compare these results for the $\Delta B=2$, six-quark n-$\bar{n}$ transition matrix elements, as calculated in the various quark confinement models, with similar calculations of the $\Delta S=2$, four-quark $K^\circ \leftrightarrow \bar{K}^\circ$ transition matrix elements[10] and the $\Delta S=0$, four-quark operators contributing to twist-four effects in electroproduction[11]. In each case, the four-quark matrix elements were also found to be quite model dependent, in contrast to the two-quark hadron matrix elements which seem to be relatively less dependent on the hadron wave functions. Since, to some extent, these latter two-quark matrix elements are used in fitting certain data to determine the various model parameters, any independent experimental information about the matrix elements of the four-quark and six quark operators would be very useful.

To summarize, we have found the values of the n-$\bar{n}$ matrix elements of the $\Delta B=2$, six-quark transition operators to be very sensitive to the quark confinement model used for the neutron wave function as shown in Tables I and II. This model dependence introduces a considerable uncertainty in any theoretical estimates of the free neutron oscillation time $\tau_{n\bar{n}}$. While this is perhaps not so surprising, given our present understanding of quark confinement, our calculations make the point quite clear, quantitatively.

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	RPM[8] $(10^{-5}\,GeV^6)$	HOSM[9] $(10^{-5}\,GeV^6)$	MIT (A)[7] $(10^{-5}\,GeV^6)$	MIT (B)[7] $(10^{-5}\,GeV^6)$
I_a	-0.842	-0.060	-0.151	-0.149
I_b	0.143	0.004	0.122	0.073
I_c	-2.138	-0.078	-0.274	-0.222

Table I: Numerical values of the integrals $I_{a,b,c}$ calculated in the Relativistic Potential Model (RPM) and the Harmonic Oscillator Shell Model (HOSM). For comparison, the values found in the MIT Bag Model for two fits to the bag parameters are also given (ref. 5).

	RPM[8] (10^{-5} GeV^6)	HOSM[9] (10^{-5} GeV^6)	MIT (A)[7] (10^{-5} GeV^6)	MIT (B)[7] (10^{-5} GeV^6)
$\langle O_1\rangle_{RRR}$	-23.64	-1.54	-6.56	-5.33
$\langle O_1\rangle_{LLR}$	-21.35	-1.47	-4.61	-4.17
$\langle O_1\rangle_{RLL}$	-14.49	-1.28	1.26	-0.67
$\langle O_2\rangle_{RRR}$	5.91	0.38	1.64	1.33
$\langle O_2\rangle_{LLR}$	7.05	0.42	2.62	1.92
$\langle O_2\rangle_{RLL}$	3.62	0.32	-0.31	0.17
$\langle O_3\rangle_{RRR}$	9.85	0.64	2.73	2.22
$\langle O_3\rangle_{LRR}$	-26.86	-1.04	-3.18	-2.72
$\langle O_3\rangle_{LLR}$	23.30	0.80	2.41	2.03

Table II: Numerical values of the n-$\bar{n}$ matrix elements of the $\Delta B=2$, six-quark transition operators $(O_m)_X$ calculated in the Relativistic Potential Model (RPM) and the Harmonic Oscillator Shell Model (HOSM). For comparison the values in the MIT Bag Model for two fits to the bag parameters are also given (ref. 5).